

Notes on Dynamics of an External Cavity Semiconductor Lasers

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Abstract. Dynamics of external cavity semiconductor lasers is known to be a complex and uncontrollable phenomenon. Due to the lack of experimental studies on the nature of the external cavity semiconductor lasers, there is a need to theoretically clarify laser dynamics. The stability of laser dynamics in the present paper, is analyzed through plotting the Lyapunov exponent spectra, bifurcation diagrams, phase portrait and electric field intensity time series. The analysis is preformed with respect to applied feedback phase C_p , feedback strength η and the pump current of the laser. The main argument of the paper is to show that the laser dynamics can not be accounted for through simply a bifurcation diagram and single-control parameter. The comparison of the obtained results provides a very detailed picture of the qualitative changes in laser dynamics.

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1. Introduction

Semiconductor lasers are ubiquitous in the modern world. They are the principal source of coherent light in optical communications and ultra-fast optical processing, they are used in optical storage devices, in laser pointers, to cite only a few applications [4, 5, 6, 7]. Semiconductor lasers provide one of the best physical systems for studying nonlinear dynamic phenomena. Many researchers have investigated the chaotic behavior of several laser systems [1, 2, 3].

Semiconductor laser has many advantages, such as small size, easy integration, compactness, low cost and convenience of operation. Therefore, they are preferable to any other types of lasers in the field of optical telecommunications.

External cavity semiconductor lasers (ECSLs) are an integral part of high speed chaos based communication systems [8, 9]. Hence, ECSLs have been a subject of extensive research [10, 11, 12]. Understanding the influence of delayed optical feedback on the behavior of ECSLs is of great relevance for technological applications.

Different studies have been conducted to characterize or manipulate of ECSLs; long external cavity [12, 14] and short external cavity [15, 16, 17]. Because of advantages in the short cavity regime [15, 18], in this paper, we just focus on the short external cavity semiconductor lasers as a dynamical system. The main model in the understanding of the dynamics of these lasers is the well-known Lang-Kobayashi (LK) equations, consisting of two delay differential equations (DDEs) [21] for the complex electrical field E and the carrier number density N [19, 20]. These equations closely describe the chaotic behavior of the physical system and confirm the presence of chaotic regimes [13]. A large part of these studies have been made possible using numerical continuation methods which allow one to find and follow numerical solution to the rate equations irrespective of their stability.

Our main point here is that one is likely to miss important phenomena if one just considers a bifurcation diagram. This is why we show the behavior of the laser for a single value of the control parameter in different ways: by Lyapunov exponents and time series. This allows us to present a consistent overall picture of the dynamics at the same time. In this study parameters such as feedback phase, feedback strength, and pump current in the short external cavity are used as control parameters to obtain different dynamical regimes, including periodic and quasi periodic (QP) [22], regular pulse packages (RPP) [18], and chaotic (CH) behavior [23].

2. The Laser model

In the early 1980s, Lang-Kobayashi (LK) proposed model semiconductor lasers [24, 25]. The *LK* equations are model equations that have been used extensively in the past to describe a semiconductor laser subject to feedback from an external cavity [25] where they used DDEs with the advantage that have an infinite-dimensional phase space [21]. For the (complex) electric field E and inversion N , we write the *LK* equations

as the dimensionless and compact set of equations [12]:

$$\frac{dE}{dt} = (1 + i\alpha)NE + \eta E(t - \tau)e^{-iC_p} \quad (1)$$

$$T \frac{dN}{dt} = P - N - (1 - 2N) |E|^2. \quad (2)$$

The parameters in the above-mentioned equations describe the line width enhancement factor α , the feedback strength η , the 2π -periodic feedback phase C_p , the ratio between carrier, photon lifetime T and the pump current \mathbf{P} . Equations (1), (2) describe a semiconductor laser with external optical feedback. In these equations, the time is normalized to the cavity photon lifetime ($1ps$) and T is the ratio of the carrier lifetime ($1ns$) to the photon lifetime [18]. In this study the parameters are chosen not only to elucidate the dynamical structure but also correspond fairly well to the experimental conditions. The external round trip time τ is also normalized to the photon lifetime. The remaining parameters, however, are held fixed at $T = 1710$, $\tau = 70$, $\alpha = 5.0$ [15]. In this paper the stability of an electric field intensity $|E|^2$ is studied versus C_p , η , and \mathbf{P} . Where all the parameters are easily accessible in experiments [15, 26].

3. Stability analysis

3.1. Bifurcation diagrams

Bifurcation means a qualitative change in the dynamical behavior of a system when a parameter of the system is varied. A bifurcation diagram provides a useful insight into the transition between different types of motion that can occur as one parameter of the system alters [27]. It enables one to study the behavior of the system on a wide range of an interested control parameter. In this paper the dynamical behavior of the system is studied through plotting the bifurcation diagrams of the $|E|^2$ versus C_p , η , and \mathbf{P} as control parameters. This procedure continued by increasing the control parameters, and the new resulting points were plotted in the bifurcation diagram versus the new control parameter.

3.2. Lyapunov exponent spectrum

Lyapunov exponents and entropy measures, on the other hand can be considered as ‘dynamic’ measures of attractor complexity and they are called ‘time average’ [27]. The Lyapunov exponent λ is useful for distinguishing various orbits. Lyapunov Exponents quantify sensitivity of the system to initial conditions and give a measure of predictability. The Lyapunov exponent is a measure of the rate at which the trajectories separate one from another. A negative exponent implies that the orbits approach to a common fixed point. A zero exponent means that the orbits maintain their relative positions; they are on a stable attractor. Finally, a positive exponent implies that the orbits are on a chaotic attractor, so the presence of a positive Lyapunov exponent indicates chaos. The Lyapunov exponent is defined as follows:

Consider two nearest neighboring points in phase space at time 0 and t , with distances of the points in the i th direction $\|\delta x_i(0)\|$, and $\|\delta x_i(t)\|$, respectively. The Lyapunov exponent is then defined by the average growth rate λ_i of the initial distance,

$$\lambda_i = \lim_{t \rightarrow \infty} \frac{1}{t} \ln \frac{\|\delta x_i(t)\|}{\|\delta x_i(0)\|} \quad (3)$$

The existence of a positive Lyapunov exponent is the indicator of chaos showing neighboring points with infinitesimal differences at the initial state abruptly separate from each other in the i th direction [28]. Using the algorithm of Wolf [29], the Lyapunov exponent was calculated versus a given control parameter. Then the value of the control parameter increased a little and the Lyapunov exponent was calculated for the new control parameter. By continuing this procedure Lyapunov exponent spectrum of the system was plotted versus the control parameter.

4. Results and Discussions

The absence of employing direct mathematical methods in study of ECSLs with respect to the variations of the parameters C_p , η , and \mathbf{P} easily could be solved by considering the bifurcation diagram and Lyapunov exponent spectrum. The output intensity ($|E|^2$) dynamics in the ECSLs, has been studied by considering both the variation of each parameter (η , C_p , \mathbf{P}) and the two other (η , C_p , \mathbf{P}) as a initial condition of ECSLs. The process is divided into three categories as defined below.

A- The effect of the feedback strength variations: The effect of the increasing of the η in the laser dynamics could be divided into the two sections of low regime η (such as $\eta < 0.0605$) and high regime η (such as $\eta > 0.0727$) [15]. In the low η as it was expected $|E|^2$ dynamics reveals chaotic (CH), periodic (P1, P2, ...), and quasi periodic (QP) behaviors. By the same token in the high regime $|E|^2$ dynamics reveals a regular plus package (RPP), QP and P1 behaviors (see Fig. 1). The basic role of the η in laser dynamics is to displace of the periodic behavior windows, which can be presented by the Lyapunov exponent spectrum and the bifurcation diagrams.

B- The effect of the feedback phase variations: The optical feedback phase is of a particular cyclic nature. Starting from a certain initial status, a variation of the C_p by 2π must turn back to its initial status [15]. To illustrate the effect of C_p and its sensitive influence on the output intensity, $|E|^2$, the study should focus on the two, Low and high regimes. In the low regime of η , the output intensity $|E|^2$ presents a P1 and P2, the chaotic behavior and QP (see Fig. 2). The appearance of the regular pulse packages (RPP) and disappearing of both the chaotic behavior and higher period (P2, P3, ...) is the natural function of the output intensity $|E|^2$ in the high regime of the C_p (see Fig. 3). The previous studies predicted bifurcation up the mode 10 (such as P1, P2, ...). However, based on the result of the present study, bifurcation can not exceed 7 mode (see Fig. 4).

C- The effect of pumping current variations: The bifurcation diagram in

variations of pump current follows the threshold value with respect to the η and C_p . Where by considering the low η , as can be seen in Fig. 5a, the $|E|^2$ is initially stable with a P1 which undergoes early cascades of period-doubling to chaos. Chaotic window in bifurcation diagram varies based on the C_p , as shown in Fig. 5(a-c).

In high η , as pictured in Fig. 6a, the $|E|^2$ is initially (at first) in P1 dynamics, which by increasing the \mathbf{P} , it undergoes RPP windows. As shown in Fig. 6(a-c), threshold of RPP is directly depended to C_p with a higher and minor growth rate. At low η , variations of periodic window and the increase of the chaotic window are the results of the increase of the pump current depicted in Fig. 7(a-c). The obtained results confirmed by plotting the time and phase portrait too (Fig. 8 and Fig. 9). Fig. 8(a-b) and Fig. 9(a-b), verify the increase of the chaotic window. Fig. 8(c-d) and Fig. 9(c-d), on the other hand confirm variations of periodic window. Also based on the result the increase of the RPP windows and the disappearance of the QP in high η is the result of the increasing of the pump current (see Fig. 10 and Fig. 11). The overall picture of $|E|^2$ dynamics has been presented in Fig. 12.

5. Summery and Outlook

Previous studies demonstrated have not provided sufficient insists towards the nature of ECSLs dynamics. Thus, there is a need to theoretically clarify its dynamics [18]. As noted before, bifurcation diagram can not reveal the hidden aspects of Laser dynamics [30, 12]. Therefore, employing the Lyapunov exponent spectrum method can cover this methodological deficiency. The obtained results shed some lights on control process of the ECSLs dynamics. As a conclusions, it can be stated that η shows its influence on Laser dynamics by selecting a method of transfer from stable status to unstable status [15] such as (Periodic, QP and chaotic) behavior or (RPP, QP and P1). Therefor η can be regarded as the most important factor in stability of the Laser. Where C_p acts as a selection method in the scenario. Pump current is the other important factor in Laser dynamics, an essential factor which can influence the ECSLs dynamics from practical point of view. The use of pump current for controlling the ECSLs dynamics can also be considered a new field for further investigations.

References

- [1] Henry D, Abarbanel I, Gills Z, Liu C and Roy R 1996 *Physical Review. A* **53** 440-453.
- [2] Chizhevsky V N and Corbalán R 1996 *Physical Review E* **54** 4576-4579.
- [3] Manffra E F, Caldas I L and Viana R L 2003 *Chaos, Solitons & Fractals* **15** 327-341.
- [4] Jia-Gui W, Guang-Qiong X, Liang-Ping C and Zheng-Mao W 2009 *Optics Communications* **282** 3153-3156.
- [5] Chen H F and Liu J M 2000 *IEEE J. Quant. Electron* **36** 27-34.
- [6] Mirasso C R, Colet P and Garcia-Fernández P 1996 *IEEE Photon. Technol. Lett.* **8** 299-301.
- [7] VanWiggeren G D and Roy R 1998 *Science* **279** 1198-1200.
- [8] Kane D M and Shore K A 2005 *unlocking dynamical diversity: Optical Feedback Effect on Semiconductor lasers* (chichester: John Wiley & sons) p 307.

- [9] Torcini A, Barland S, Giacomelli G and Marin F 2006 *Physical Review A* **74** 063801-13.
- [10] Antonelli C and Mecozzi A 2009 *Optics Communications* **282** 2917-20.
- [11] Steele R 2004 *Laser Focus World* **40** 75-82.
- [12] Green K. 2009 *Physical Review E* **79** 036210-22.
- [13] Fischer I, Liu Y and Davis P 2000 *Physical Review A* **62** 011801-4.
- [14] Haegeman B, Engelborghs K, Roose D, Pieroux D and Erneux T 2002 *Physical Review E* **66** 0462161-11.
- [15] Heil T, Fischer I, Elsässer W, Krauskopf B, Green K and Gavrielides A 2003 *Physical Review E* **67** 0662141-11.
- [16] Ahmed M 2009 *Optics Laser Technology* **41** 53-63.
- [17] Peil M, Fischer I and Elsässer W 2006 *Physical Review A* **73** 0238051-13.
- [18] Heil T, Fischer I and Elsässer W 2001 *Physical Review Letter* **87** 243901-4.
- [19] Ohtsubo J 2006 *Semiconductor Lasers-Stability, Instability and Chaos* (Berlin: Springer Series in Optical Sciences, Springer-Verlag) Chapter 8.
- [20] Ruiz-Oliveras F R and Pisarchik A N 2009 *Physical Review E* **79** 016202-9.
- [21] Erneux T 2004 *Physical Review E* **69** 036210-17.
- [22] Murakami A and Ohtsubo J 1998 *IEEE J. Quantum Electron* **34** 1979-1983.
- [23] Mork J, Tromborg B and Mark J 1992 *IEEE J. Quantum Electron* **28** 93-108.
- [24] Lang R and Kobayashi K 1980 *IEEE J. Quantum Electron* **16** 347-355.
- [25] Verduyn Lunel S M and Krauskopf B 2000 *International Spring School. AIP Conf. Proc. No. 548* (New York: AIP Melville) p 66.
- [26] Mork J, Mark J and Tromborg B 1990 *Physical Review Lett* **65** 1999-2002.
- [27] Dorfman J R 1999 *An introduction to chaos in nonequilibrium statistical mechanics* (New York: Cambridge University Press) Chapter 8.
- [28] Ott E 2002 *Chaos in dynamical system* (New York: Cambridge University Press) Chapter 4.
- [29] Wolf A, Swift J B, Swinney H L and Vastano A 1985 *Physica D* **16D** 285-317.
- [30] Krauskopf B, Gray G R and Lenstra D 1998 *Physical Review E* **58** 7190-7.

Figure Captions

Fig.1. Development of the dynamics for $|E|^2$ ($\mathbf{P} = 0.8$ and $C_p = 1$) (a)- Bifurcation diagram (b)-Lyapunov exponent spectrum.

Fig.2. Illustration of the effect of the C_p variation on $|E|^2$ ($\mathbf{P} = 0.8$ and $\eta = 0.036$) (a)- Bifurcation diagram (b)-Lyapunov exponent spectrum.

Fig.3. Illustration of the effect of the C_p variation on $|E|^2$ ($\mathbf{P} = 0.8$ and $\eta = 0.090$) (a)- Bifurcation diagram (b)-Lyapunov exponent spectrum.

Fig.4. A comparative description of various periodic modes ($\mathbf{P} = 0.8$) (a)- $\eta = 0.042$, (b)- $\eta = 0.0455$, (c)- $\eta = 0.048$.

Fig.5. The analysis of $|E|^2$ dynamics ($\eta = 0.0455$), (a)- $C_p = -1$, (b)- $C_p = 0$, (c)- $C_p = 1$.

Fig.6. The analysis of $|E|^2$ dynamics ($\eta = 0.115$), (a)- $C_p = -1$, (b)- $C_p = 0$, (c)- $C_p = 1$.

Fig.7. The role of pump currents as initial condition in periodic and chaotic windows ($\eta = 0.0455$) (a), (b) $\mathbf{P} = 0.6$; (c), (d) $\mathbf{P} = 0.7$; (e), (f) $\mathbf{P} = 1.4$.

Fig.8. Time series of $|E|^2$ ($\eta = 0.0455$) (a)- $C_p = -1$ and $\mathbf{P} = 0.6$ (QP), (b)- $C_p = -1$ and $\mathbf{P} = 0.7$ (Chaotic), (c)- $C_p = -2.835$ and $\mathbf{P} = 0.7$ (P4), (d)- $C_p = -1$ and $\mathbf{P} = 1.4$ (P2).

Fig.9. Phase diagram of $|E|^2$ ($\eta = 0.0455$) (a)- $C_p = -1$ and $\mathbf{P} = 0.6$ (QP), (b)- $C_p = -1$ and $\mathbf{P} = 0.7$ (Chaotic), (c)- $C_p = -2.835$ and $\mathbf{P} = 0.7$ (P4), (d)- $C_p = -1$ and $\mathbf{P} = 1.4$ (P2).

Fig.10. The effect of \mathbf{P} in increasing the RPP windows ($\eta = 0.042$) (a)- $\mathbf{P} = 0.8$ (b)- $\mathbf{P} = 1.4$.

Fig.11. Increasing RPP windows on $|E|^2$ ($C_p = 0$, $\eta = 0.09$ and $\mathbf{P} = 0.8$) (a)- Time series (b)- Phase diagram.

Fig.12. The overall picture of $|E|^2$ dynamics.

































































